# Optimization of LoRaWAN planning

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LPWAN days, Lyon

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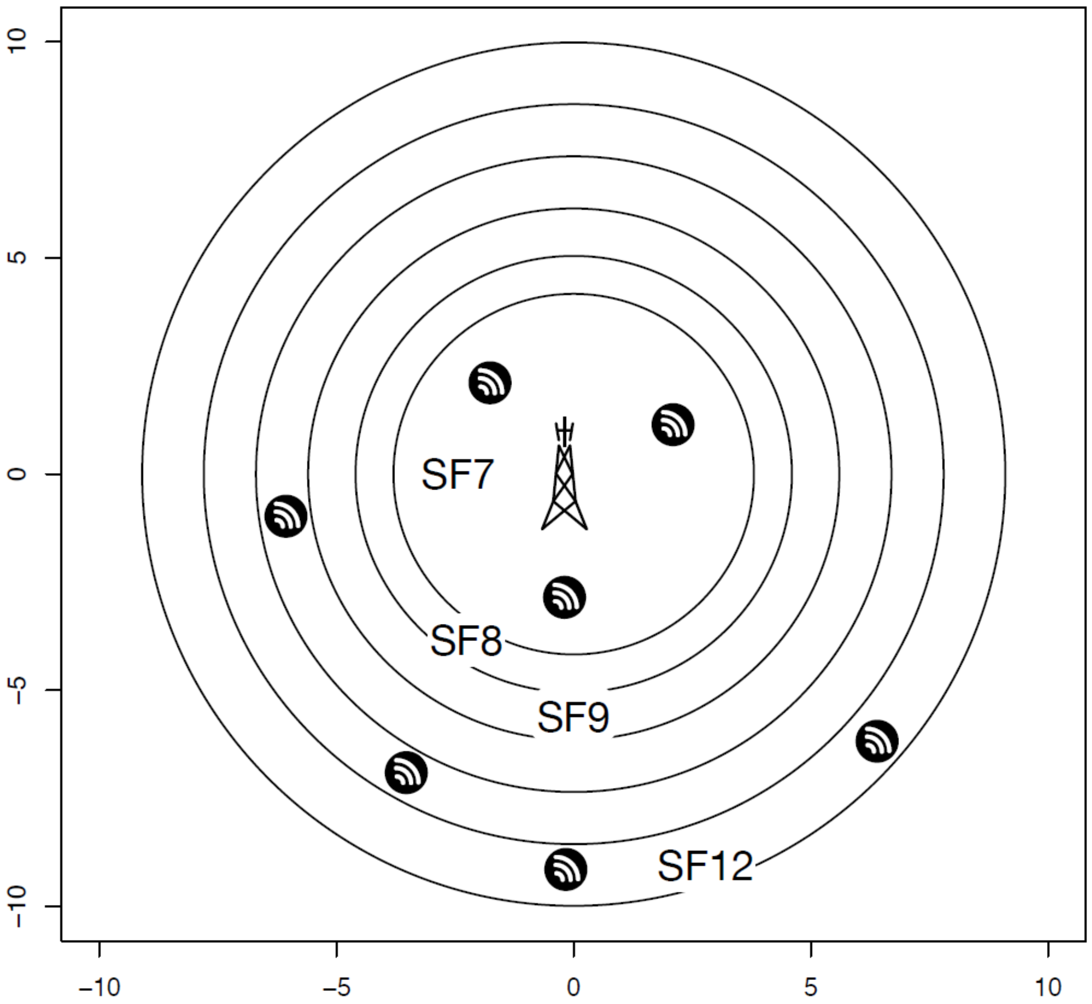
# Capacity of a LoRaWAN cell

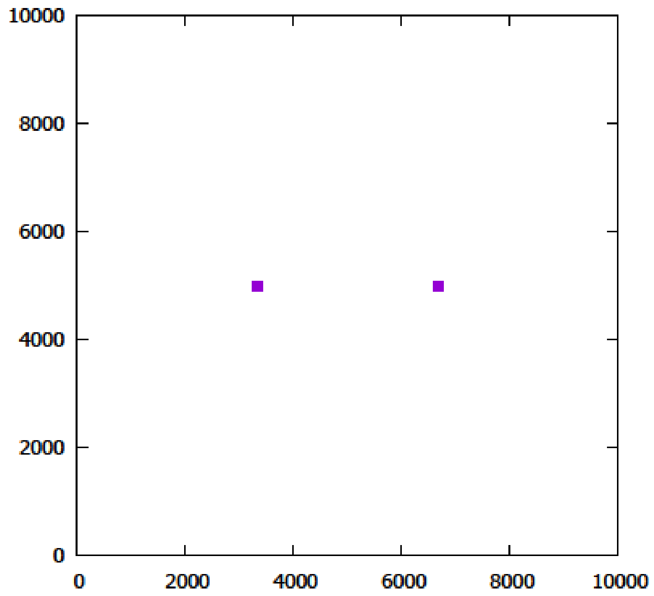
- Analytical computation of the single cell capacity in terms of maximum number of nodes

- Optimization of SF boundaries using Nelder-Mead simplex heuristic algorithm for non linear models

- Study of node density to optimize the SF boundaries

➔ Optimization of LoRaWAN planning with one or several gateways to maximizing the theoretical capacity


SF7
SF8
SF9
SF12
10
5
0
−5
−10
−10
−5
0
5
10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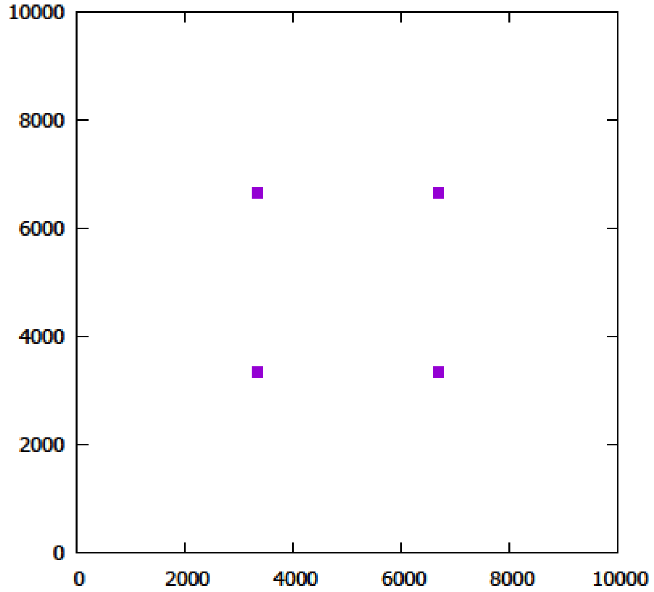
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# Related work

- D. Zorbas, G. Z. Papadopoulos, P. Maillé, N. Montavont, and C. Douligeris, ICT 2018
  - One gateway, all nodes can use all SFs
  - Find proportion of nodes in each SF (only 7 and 8)

- M. Cesana, A. Redondi, and J. Ortin, PIMRC 2018
  - Multiple gateways but each node associated with closest gateway
  - Performance metric DER and delay

# Optimization model

- **Goal** : maximize the number of served nodes in LoRaWAN

- Served if :
    1. Probability of successfull transmission above threshold $\beta$

$$H = \exp\left(-\frac{Nq_j}{Pg(d)}\right) \geq \beta$$

Transmission successfull if the SNR at the receiver is above the min SNR $q_j$ for DRj at SF=12-j

Signal power depends on distance, Rayleigh fading, Okumura Hata path loss attenuation

# Optimization model

- **Goal** : maximize the number of served nodes in LoRaWAN

- Served if :
  2. Success probability among concurrent transmissions above threshold $\gamma$

$$\Pr(i) = e^{(-2T^f_{pkt}\lambda N_i)} \geq \gamma$$

None of potential interferers Ni transmits within $2T^f_{pkt}$ (airtime at SF f) based on un-slotted ALOHA

# Capture effect

- Collision may occur between 2 simultaneous frames in same BW using same SF
- But frame with higher received power can be decoded if at least 6dB more

➔ Intra-SF interference parameter between nodes i and j in SF f :

$$C_{ij}^{f} = \begin{cases} 1 & \text{if } P_{rx}^{i} - P_{rx}^{j} \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

Multiple gateways : 1 if power difference less than 6dB at all gateways

# Imperfect SF orthogonality

- 2 frames using different SFs can collide if received power significantly different at the gateway

TABLE I: SINR thresholds for co-SF interference

| interferer / desired | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|
| 7 | 6 | -16 | -18 | -19 | -19 | -20 |
| 8 | -24 | 6 | -20 | -22 | -22 | -22 |
| 9 | -27 | -27 | 6 | -23 | -25 | -25 |
| 10 | -30 | -30 | -30 | 6 | -26 | -28 |
| 11 | -33 | -33 | -33 | -33 | 6 | -29 |
| 12 | -36 | -36 | -36 | -36 | -36 | 6 |

C. Goursaud J-M. Gorce, EAI Trans. On IoT, 2015

➔Inter-SF interference parameter between nodes i at SF f and j at SF f' :

$$I_{ij}^{ff'} = \begin{cases} 1 & \text{if } P_{rx}^{i} - P_{rx}^{j} \leq SINR_{ff'} \\ 0 & \text{otherwise} \end{cases}$$

Multiple gateways : 1 if power difference less than SINR at all gateways

# SF allocation problem

- Input
  - Set of deployed nodes I
  - Set of potential interferers Ni for each node i
  - Threshold $\beta$ and $\gamma$

- Output
  - SF allocation for served nodes maximizing their cardinality

# Nonlinear program

$$\max \sum_{i \in \mathcal{I}} \sum_{f \in \{7,\dots,12\}} \omega_i y_i^f \tag{5}$$

$$\sum_{f \in \{7,\dots,12\}} y_i^f \leq 1 \qquad \forall i \in \mathcal{I} \tag{6}$$

$$H \geq \beta y_i^f \quad \forall i \in \mathcal{I}, f \in \{7, \dots, 12\} \tag{7}$$

$$e^{-2\lambda \sum_{f \in \{7,\dots,12\}} T_{pkt}^f y_i^f (1 + \sum_{j \neq i} C_{ij}^f y_j^f + \sum_{f' \neq f} \sum_{j \neq i} I_{ij}^{ff'} y_j^{f'})} \geq \gamma \quad \forall i \in \mathcal{I} \tag{8}$$

- Max served nodes based on weight wi
- One node : at most one SF
- SNR threshold to allow SF allocation
- Success proba guaranty

# Nonlinear program

$$\max \sum_{i\in\mathcal{I}} \sum_{f\in\{7,\dots,12\}} \omega_i y_i^f \qquad (5)$$

$$\sum_{f\in\{7,\dots,12\}} y_i^f \le 1 \qquad \forall i \in \mathcal{I} \qquad (6)$$

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$$e^{-2\lambda \sum_{f\in\{7,\dots,12\}} T_{pkt}^f y_i^f (1+\sum_{j\neq i} C_{ij}^f y_j^f + \sum_{f'\neq f}\sum_{j\neq i} I_{ij}^{ff'} y_j^{f'})} \ge \gamma \qquad \forall i \in \mathcal{I} \qquad (8)$$

- Max served nodes based on weight wi
- One node : at most one SF
- SNR threshold to allow SF allocation
- Success proba guaranty

In case of multiple gateways, we take the maximum H over all gateways for a particular SF (meaning that the node can reach at least 1 gateway using the SF if max $H \geq \beta$ )

# Nonlinear program

$$\max \sum_{i \in \mathcal{I}} \sum_{f \in \{7,\ldots,12\}} \omega_i y_i^f \qquad (5)$$

$$\sum_{f \in \{7,\ldots,12\}} y_i^f \leq 1 \qquad \forall i \in \mathcal{I} \qquad (6)$$

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$$e^{-2\lambda \sum_{f \in \{7,\ldots,12\}} T_{pkt}^f y_i^f (1 + \sum_{j \neq i} C_{ij}^f y_j^f + \sum_{f' \neq f} \sum_{j \neq i} I_{ij}^{ff'} y_j^{f'})} \geq \gamma \qquad \forall i \in \mathcal{I} \qquad (8)$$

- Max served nodes based on weight wi
- One node : at most one SF
- SNR threshold to allow SF allocation
- Success proba guaranty
  - ➔ But SFs are not known !

# ~~Non~~linear program

$$e^{-2\lambda \sum_{f \in \{7,\dots,12\}} T_{pkt}^{f} y_i^f (1+\sum_{j \neq i} C_{ij}^{f} y_j^f + \sum_{f' \neq f} \sum_{j \neq i} I_{ij}^{ff'} y_j^{f'})} \geq \gamma \qquad \forall i \in \mathcal{I} \qquad (8)$$

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# ~~Non~~linear program

$$e^{-2\lambda \sum_{f\in\{7,\ldots,12\}} T^f_{pkt} y^f_i (1+\sum_{j\neq i} C^f_{ij} y^f_j + \sum_{f'\neq f} \sum_{j\neq i} I^{ff'}_{ij} y^{f'}_j)} \geq \gamma \qquad \forall i \in \mathcal{I} \qquad (8)$$

$$\Downarrow$$

$$\sum_{f} T^f_{pkt} y^f_i (1+\sum_{j\neq i} C^f_{ij} y^f_j + \sum_{f'\neq f} \sum_{j\neq i} I^{ff'}_{ij} y^{f'}_j) \leq -\frac{\log(\gamma)}{2\lambda}$$

# ~~Non~~linear program

$$\sum_{f} T^{f}_{pkt} y^{f}_{i} (1 + \sum_{j \neq i} C^{f}_{ij} y^{f}_{j} + \sum_{f' \neq f} \sum_{j \neq i} I^{ff'}_{ij} y^{f'}_{j}) \leq -\frac{\log(\gamma)}{2\lambda}$$

# ~~Non~~linear program

$$\sum_f T^f_{pkt} y^f_i (1 + \sum_{j \neq i} C^f_{ij} y^f_j + \sum_{f' \neq f} \sum_{j \neq i} I^{ff'}_{ij} y^{f'}_j) \leq -\frac{\log(\gamma)}{2\lambda}$$

- If $y^f_i$ = 0 : no limit for the node at SF f

# ~~Non~~linear program

$$\sum_f T^f_{pkt} y^f_i \left(1 + \sum_{j \neq i} C^f_{ij} y^f_j + \sum_{f' \neq f} \sum_{j \neq i} I^{ff'}_{ij} y^{f'}_j\right) \leq -\frac{\log(\gamma)}{2\lambda}$$

- If $y^f_i$ = 0 : no limit for the node at SF f

- If $y^f_i$ = 1 : $T^f_{pkt}\left(1 + \sum_{j \neq i} C^f_{ij} y^f_j + \sum_{f' \neq f} \sum_{j \neq i} I^{ff'}_{ij} y^{f'}_j\right) \leq -\frac{\log(\gamma)}{2\lambda}$ should hold

# ~~Non~~linear program

$$\sum_{f} T^f_{pkt} y^f_i (1 + \sum_{j \neq i} C^f_{ij} y^f_j + \sum_{f' \neq f} \sum_{j \neq i} I^{ff'}_{ij} y^{f'}_j) \leq -\frac{\log(\gamma)}{2\lambda}$$

- If $y^f_i$ = 0 : no limit for the node at SF f
- If $y^f_i$ = 1 : $T^f_{pkt}(1 + \sum_{j \neq i} C^f_{ij} y^f_j + \sum_{f' \neq f} \sum_{j \neq i} I^{ff'}_{ij} y^{f'}_j) \leq -\frac{\log(\gamma)}{2\lambda}$ should hold

$$T^f_{pkt}(1 + \sum_{j \neq i} C^f_{ij} y^f_j + \sum_{f' \neq f} \sum_{j \neq i} I^{ff'}_{ij} y^{f'}_j) \leq -\frac{\log(\gamma)}{2\lambda} + M(1 - y^f_i), \ \forall i \in \mathcal{I}, f \in \{7, ..., 12\} \quad (9)$$

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# Scenario

TABLE II: Numerical values for a $\lambda = 59$ byte packet at $BW = 125\,\text{kHz}$

| SF $f$ | Time on air $T^f_{pkt}$ (ms) | Required SNR $q_f$ (dB) |
|---|---|---|
| 7 | 102 | -6 |
| 8 | 184 | -9 |
| 9 | 328 | -12 |
| 10 | 616 | -15 |
| 11 | 1 315 | -17.5 |
| 12 | 2 466 | -20 |

- 10km*10km area
- 1, 2, or 4 gateways
- 50-1000 uniformly deployed nodes

- Ensure H above $\beta$ = 66%
- Ensure Pr(i) above $\gamma$ = [50, 70, 85, 95%]
- Traffic intensity $\lambda = \frac{1}{747\,s}$ maximum at SF12 for 59B frames at their duty cycle limit

- LP solved by Cplex (time limit 1h for more than 500 nodes)

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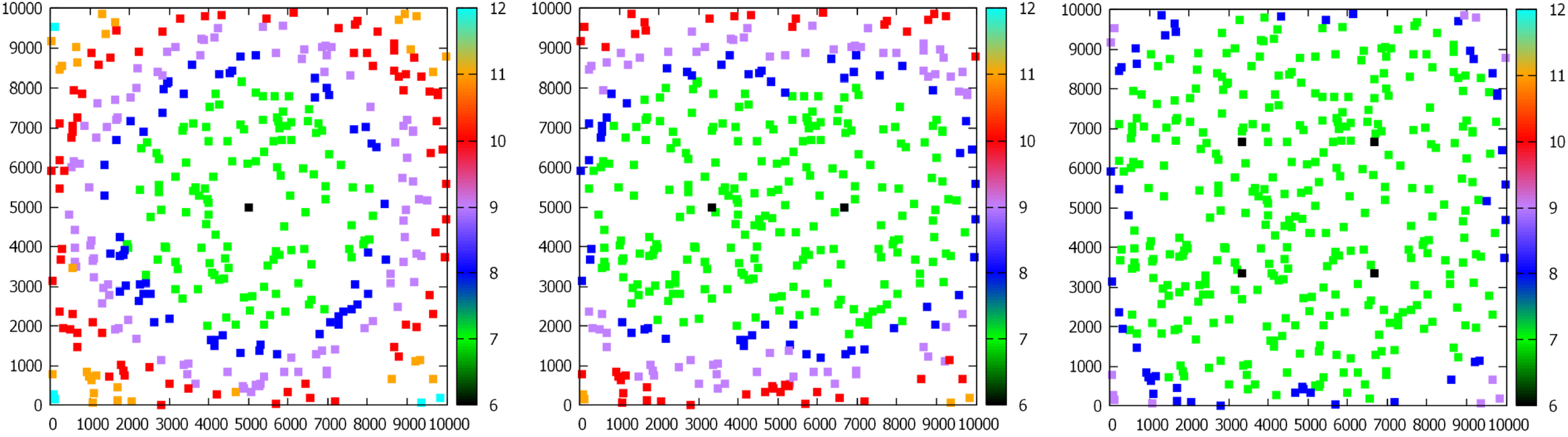
# Results




(a) 1 gateway, 387 of 400 nodes served

(b) 2 gateways, all 400 nodes served

(c) 4 gateways, all 400 nodes served

Fig. 4: Optimal SF allocation for a network of $N = 400$ and $\gamma = 50\%$.

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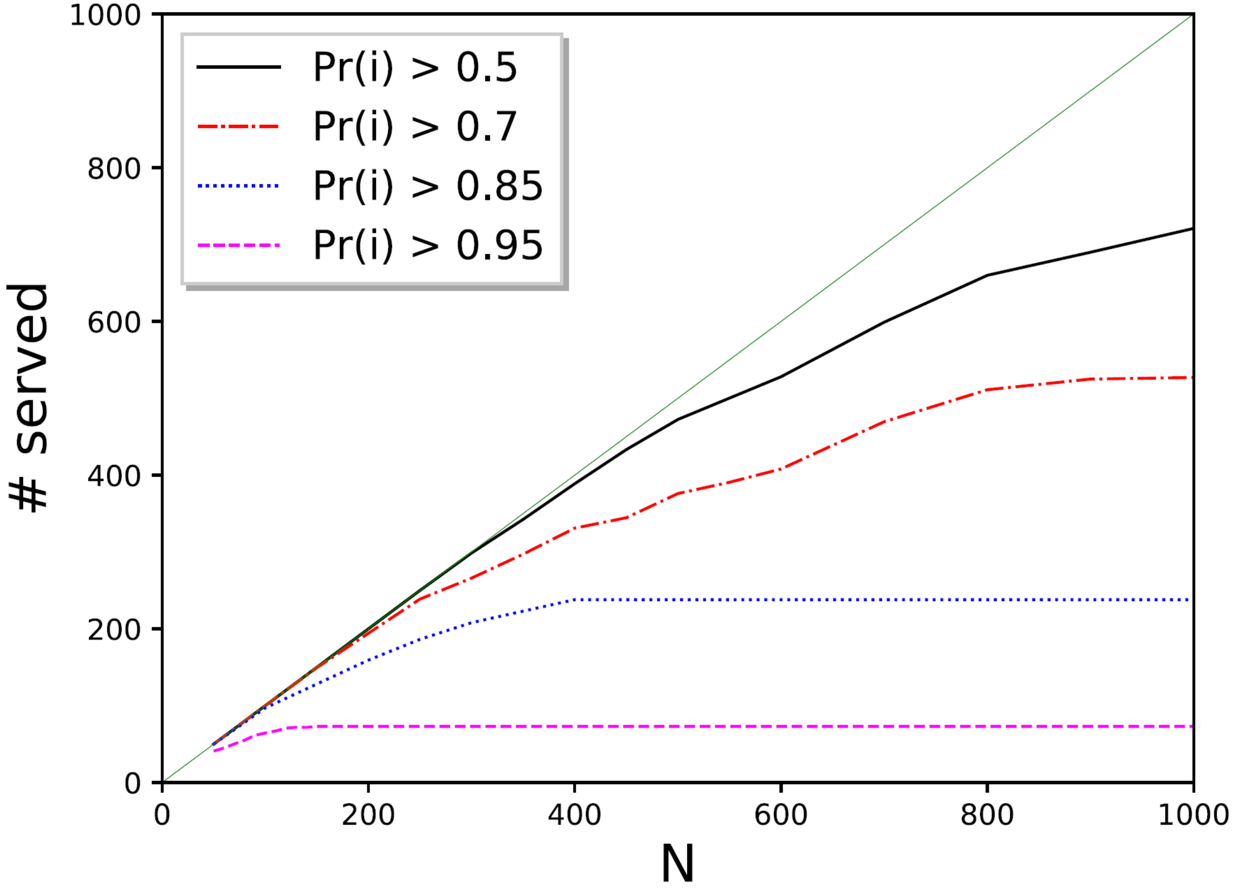
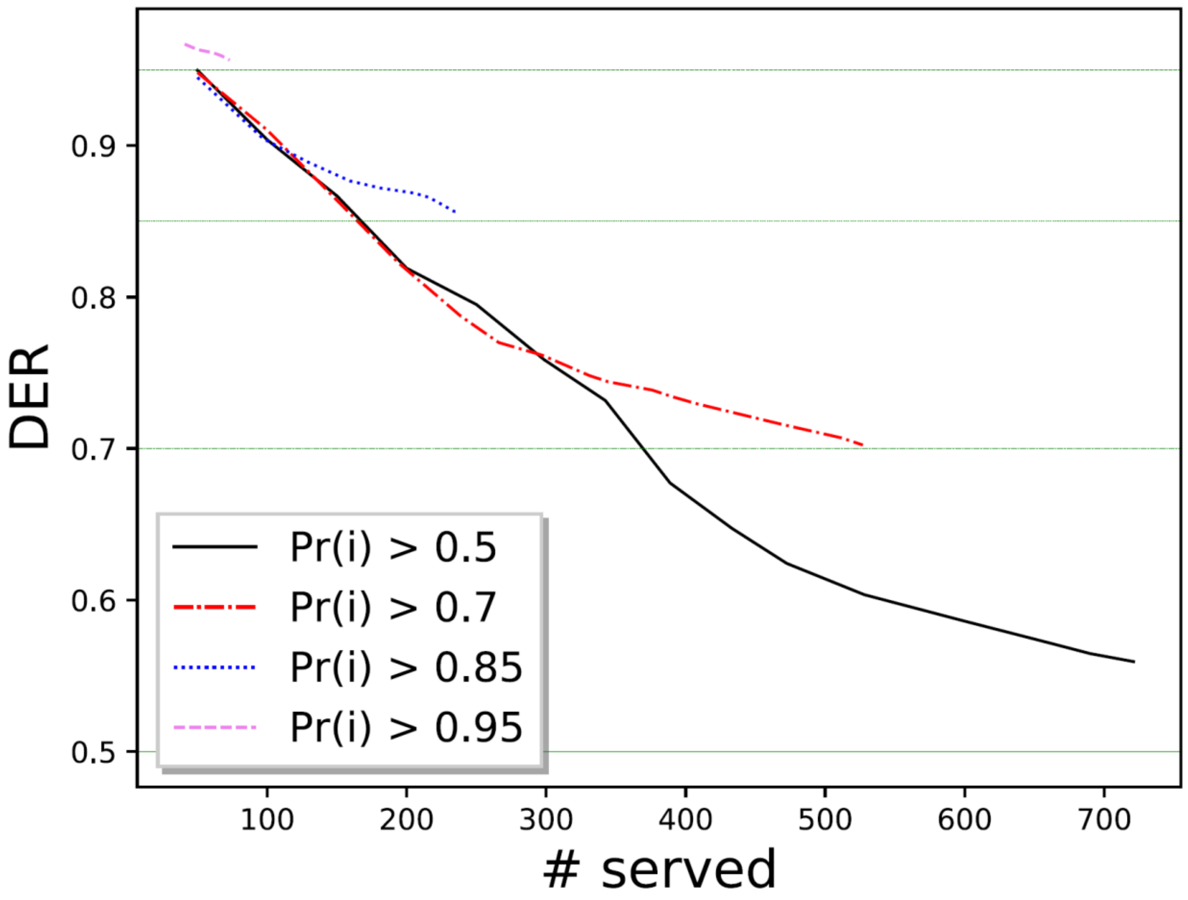
# Single cell analysis




$$\mathrm{DER} = e^{(-2N_f \cdot T_{pkt}^{f} \cdot \lambda)}$$

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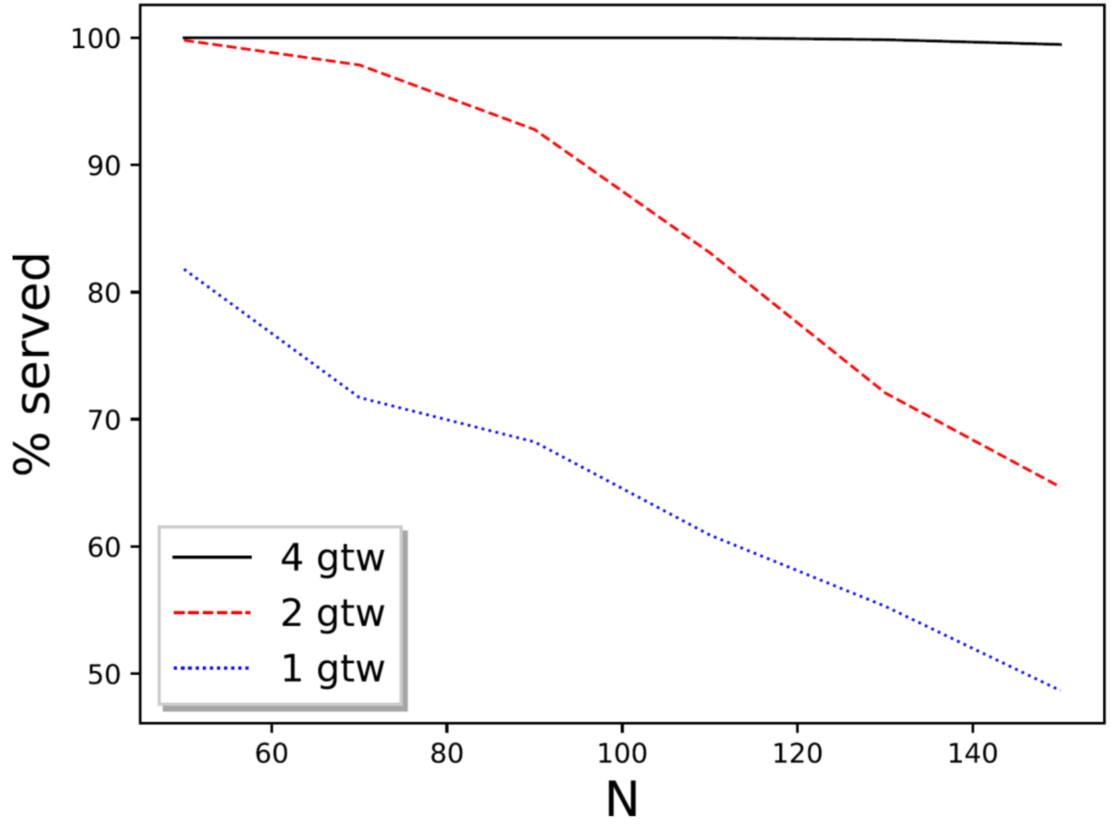
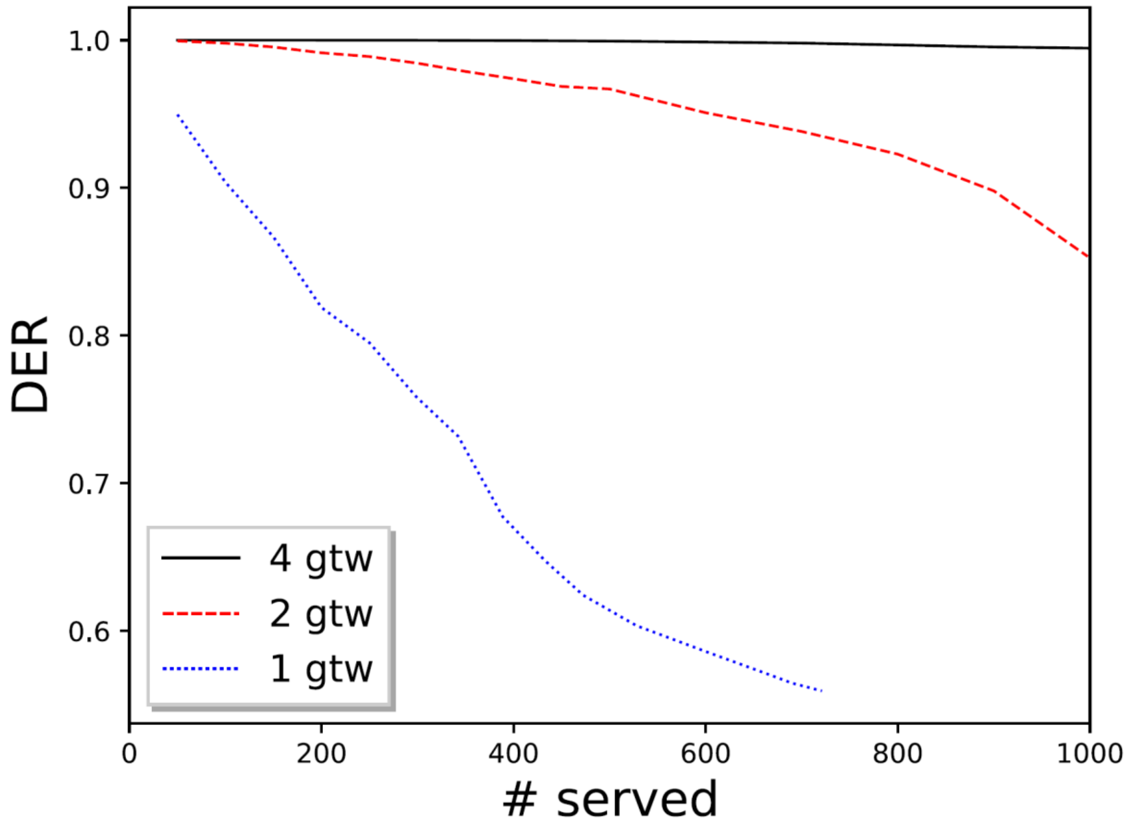
# Multiple gateway gain




$$\mathrm{DER}_{\mathrm{NetServer}} = 1 - \prod_{g \in \mathcal{G}} (1 - \mathrm{DER}_g)$$

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# Conclusions

- Optimal model for SF allocation maximizing the number of served nodes in LoRaWAN
  - Ensuring SNR conditions and probability of successfull transmission at the receiver
  - Considering potential physical capture at the receiver and imperfect SF orthogonality
- Analysis of the maximum number of served nodes in a single cell
- Quantify the gain of using multiple gateways

# On-going work

- Combine probabilities to define actual PDR of each node

- Introduce PDR as the metric for node SF allocation

Objective : $\max(\min(PDR))$

where $PDR = H \times Q$ computed for each node
and Q is the probability of successfull transmission in isolated case and with physical capture

- Use column generation to help solving larger instances