Spatial Issues in Modeling LoRaWAN Capacity

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JT Rescom 2019

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Overview

1. Models for LoRaWAN capacity
2. Motivation for nonuniform node density
3. LoRaWAN capacity for nonuniform node density

Part 1: Models for LoRaWAN capacity

signals since the system is assumed ergodic (i.e., any two instances of time are statistically independent). Note that the transmit powers of end-devices with the same SF signals are assumed equal. The second outage condition is therefore given by the complement of:

$$Q_1 = \mathbb{P}\Big[\frac{|h_1|^2 g(d_1)}{|h_{k^*}|^2 g(d_{k^*})} \geq 4 \Big| d_1\Big], \tag{4}$$

thus providing a statistically meaningful performance metric quantifying when collisions of the same SF are significant. Intuitively, we expect Q_1 to decay with increasing $\bar{N}$.

Combined, the two outage conditions form the joint outage probability J_1 of a received signal s_1 given by the complement of a successfully received signal defined as $J_1 = 1 - H_1 Q_1$.

3) Coverage Probability: The coverage probability is the probability that a randomly selected end-device is in coverage (i.e., not in outage) at any particular instance of time. One may obtain the system's coverage probability $\wp_c$ with respect

given by $f_{d_i}(x) = 2\pi x / |\mathcal{V}(d_1)|$. Calculating the pdf of $g(d_i)$

$$f_{g(d_i)}(x) = \Big|\frac{\mathrm{d}}{\mathrm{d}x} g^{-1}(x)\Big| f_{d_i}(g^{-1}(x)) = \frac{\lambda^2 x^{-\frac{\eta+2}{\eta}}}{8\eta\pi|\hat{\mathcal{V}}(d_1)|} \tag{8}$$

which has a finite support on $g(l_{j+1}) \leq x \leq g(l_j)$, and recalling that $|h_i|^2 \sim \exp(1)$, it follows that the pdf of X_i is

$$\begin{aligned} f_{X_i}(z) &= \int_{g(l_{j+1})}^{g(l_j)} \frac{1}{x} f_{g(d_i)}(x) f_{|h_i|^2}(z/x) \mathrm{d}x \\ &= \frac{\lambda^2 z^{-\frac{\eta+2}{\eta}}}{8\eta\pi|\hat{\mathcal{V}}(d_1)|} \Big[\Gamma\Big(1 + \frac{2}{\eta}, \frac{z}{g(x)}\Big)\Big]_{x=l_{j+1}}^{x=l_j}, \end{aligned} \tag{9}$$

supported on $z \in \mathbb{R}^+$, where $\Gamma(\cdot,\cdot)$ is the upper incomplete gamma function. Integrating (9) we arrive at the cdf of X_i

$$F_{X_i}(z) = \frac{z^{\frac{2}{\eta}} \lambda^2}{16\pi|\hat{\mathcal{V}}(d_1)|} \Big[\frac{(e^{\frac{-z}{g(x)}} - 1) z^{\frac{2}{\eta}}}{g(x)^{\frac{2}{\eta}}} - \Gamma\Big(1 + \frac{2}{\eta}, \frac{z}{g(x)}\Big)\Big]_{x=l_{j+1}}^{x=l_j} \tag{10}$$

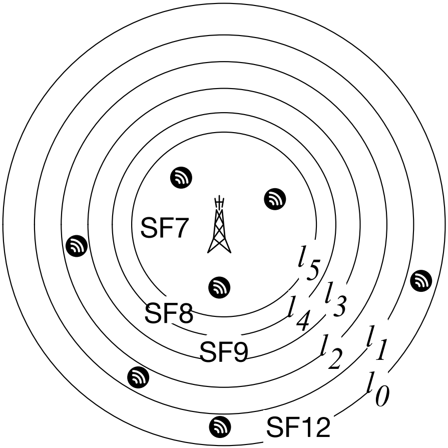
Models for LoRaWAN capacity

- **How many nodes** can a single GW handle?
  - We are looking at **uplink capacity** only!
- LoRaWAN operation
  - **Aloha** access
    - With **physical capture**!

      Reception if the colliding frame is 6 dB weaker
  - Several spreading factors SF7 — SF12
    - **Quasi-orthogonal** symbols (9 to 25 dB rejection)
    - Transmission duration $\sim$ doubles from SF_n to SF_{n+1}
  - Stringent **duty cycle** limitations (**1%** in each sub-band)
  - Long transmission times for high SF

    2.5 s of time on air at **SF12** for **59 bytes**!

SF boundaries



LoRaWAN capacity models

Low Power Wide Area Network Analysis: Can LoRa Scale?
O. **Georgiou** and U. **Raza**
IEEE Wireless Communications Letters **2017**

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⇑ Check out the cool math formulas! ⇑

Georgiou model

Outage due to attenuation and fading:

$$H(l_j) = \exp\left(-\frac{Nq_j}{Pg(l_j)}\right), \tag{1}$$

P - transmission power, N - in-band noise power,
q_j - SNR threshold, $g(l_j)$ - channel gain at l_j

Outage due to collisions

$$Q_1(l_j, v_j) = \frac{2\exp(-2v_j){l_j}^{\eta}(\eta+2)S_j}{\pi 2 v_j {l_j}^{\eta+2} + {l_j}^{\eta}(2(\eta+2)S_j - 2\pi v_j {l_j}^2)}, \tag{2}$$

η - path loss exponent, v_j - traffic occupancy,
S_j - surface of annulus j

Packet Delivery Ratio:

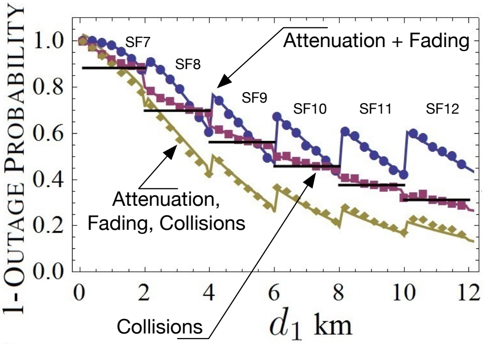
$$PDR(l_j, v_j) = H(l_j) \times Q_1(l_j, v_j) \tag{3}$$

Georgiou model with different assumptions

Changes:

- 3 channels per band $\Rightarrow$ duty cycle: **0.33%**
- Realistic application traffic - use the **same traffic for all SF**:
  - Saturate SF12: 59B, 2.466 s of time on air, 1 packet / 747 s per frequency channel
- Double traffic intensity v_j to reflect correctly the behavior of unslotted ALOHA

Example of Georgiou model results



500 nodes

- takes into account capture effect
- $H(l_j)$ — Outage due to attenuation and fading
- $Q_1(l_j, v_j)$ — Outage due to collision

LoRaWAN capacity models

How Many Sensor Nodes Fit In A LoRAWAN Cell?
M. **Heusse** et al.
Submitted **2019**

is more realistic, but the results are qualitatively similar. We consider a GW-side antenna gain of 6 dB which compensates for a receiver noise factor of 6 dB.

We neglect shadowing, since its net effect would be small in our case: it would modify the channel gain from each node without changing the general behavior of the system [6].

Finally, we consider a Rayleigh channel, so that the received signal power is affected by a multiplicative random variable with an exponential distribution of unit mean.

B. Frame reception with no interference

Provided that there is no collision, a frame transmission succeeds as long as the SNR at the receiver for this transmission is above q_j, the minimum SNR for the corresponding spreading factor [10]. The signal power depends on the distance and Rayleigh fading, whereas the noise power is the constant thermal noise for a 125 kHz-wide band: $N = -123\,\text{dBm}$ (-174 dBm per Hz). We use a transmission power $P = 14\,\text{dBm}$.

Thus, the probability of successful transmission from distance d at data rate DR*j* is:

$$H = \exp\left(-\frac{Nq_j}{Pg(d)}\right), \tag{1}$$

where $g(d)$ is the average channel gain at distance d [1].

For a tagged transmission, the probability of case 1 is $Q_1 = \exp(-2v_j)$, as there should be no other transmission event during $2\tau_j$ to avoid overlap. Case 2 happens when a single transmission occurs during this time with probability $2\,v_j \exp(-2v_j)$. If we neglect the variability of $g(d)$ among the nodes using the same SF, the received power in each annulus follows an exponential distribution. Successful frame capture occurs for a difference of 6 dB in capture effect, which means a factor of 4. Since the probability that an exponential random variable is x times above another one is $\frac{1}{x+1}$, the success probability of frame capture is $\frac{1}{5}$ and the probability of success in case 2 is

$$Q_2 = \frac{2}{5}\,v_j \exp(-2v_j). \tag{2}$$

Thus, the presence of concurrent traffic impacts the packet reception probability by a ratio Q:

$$Q = Q_1 + Q_2 = (1 + \frac{2}{5}v_j)\exp(-2v_j). \tag{3}$$

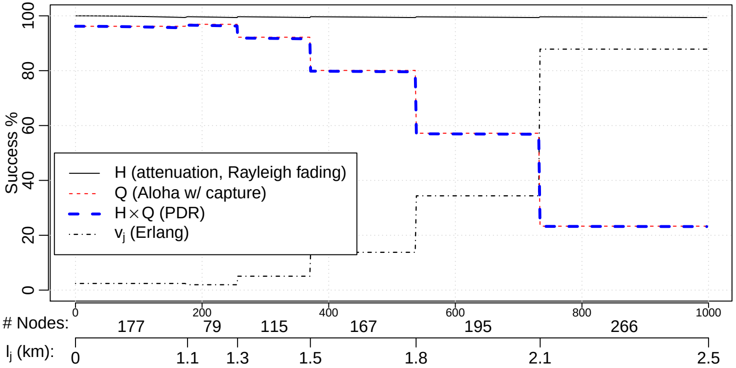
And, combining 1 and 3, we get the probability of successful packet reception:

$$PDR = H \times Q \tag{4}$$

D. Traffic intensity

⇑ Check out the elegant math formulas! ⇑

Heusse model



1000 nodes

- H — Outage due to attenuation and fading
- Q — Outage due to collision (fits Giorgiou expression for Q_1)
- v_j — Traffic occupancy

Node placement

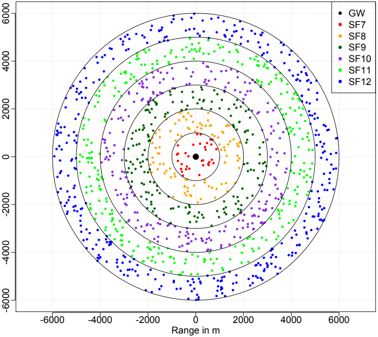
- All models assume **uniform density**
  - devices are located at random in the annulus at l_j according to a Poisson Point Process (PPP) with intensity ρ
  - no devices beyond l_0
- Number of nodes proportional to **surface** S_j of annulus j

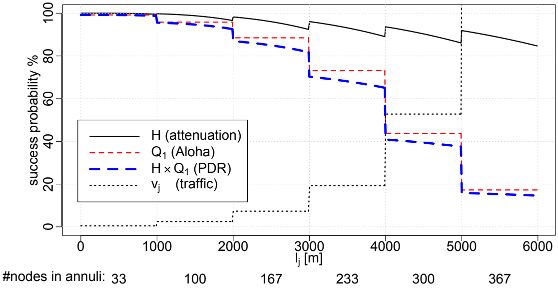
Equidistant SF boundaries [km]

	SF7	SF8	SF9	SF10	SF11	SF12
	l_5	l_4	l_3	l_2	l_1	l_0
	1	2	3	4	5	6
S_j/π [km^2]	1	3	5	7	9	11

S_j/π [km^2]: value proportional to the number of devices.

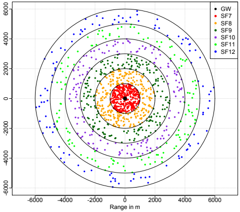
Uniform density





Uniform density and **equidistant SF annuli**, $n = 1200$.
300 nodes with PDR $> 80\%$.

Part 2: Motivation for nonuniform node density



Network planning in HetNets

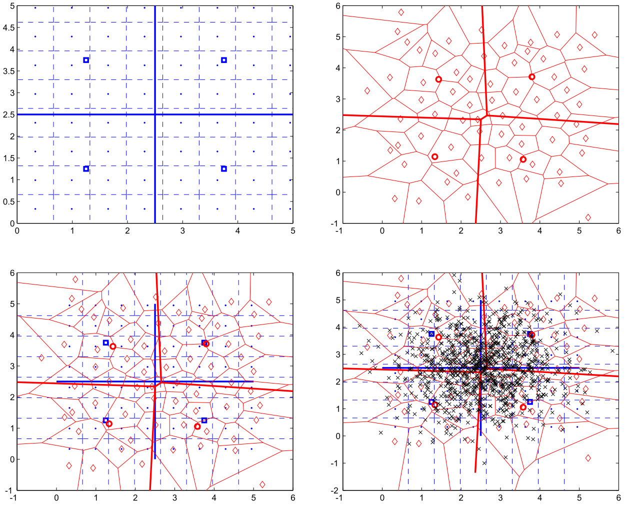
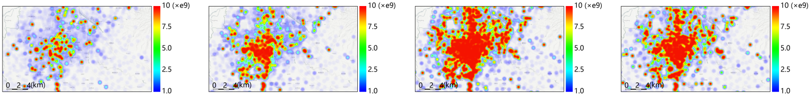


FIGURE 7. BS deployment (HetNet: 4 macro BSs and 64 small cell BSs) for a Gaussian user distribution over 25 km^2 with a density of 40 users/km^2. Upper left: Initial deployment. Upper right: Optimized deployment (Simulated Annealing). Lower left: Superposition of the two deployments. Lower right: deployments with the distribution of user terminals in the network (black 'x"s).

Cellular traffic measurements



(a) 04:00 (b) 10:00 (c) 16:00 (d) 00:00

Fig. 2: Spatial distribution of urban cellular traffic at different times of a day.

- 1.5 million users and 5,929 cell towers in a major city of China

Reasons for nonuniform density in LoRa

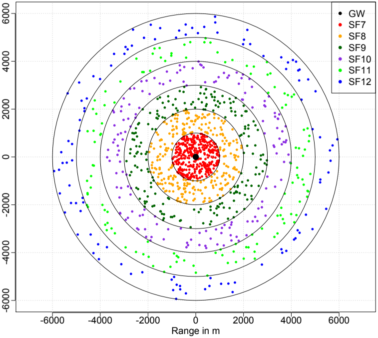
- Cellular networks: spatial traffic distribution is **highly nonuniform** across different cells
- Similar pattern to population and building densities in cities: **density decreases** with the distance from the center
- LoRa deployments: place networks close to potential users to **create hot spots near high density areas**.
- **Distance-discouraging effect**: large SF (e.g., SF11 or SF12) imply long transmission times, so increased contention (more collisions) and higher energy consumption.

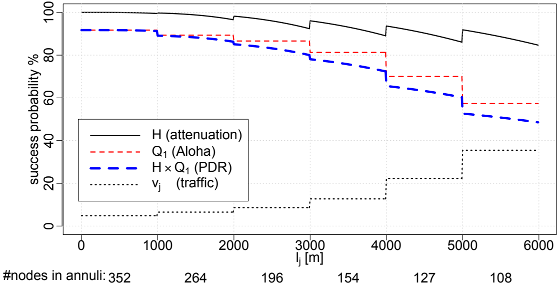
Nonuniform density

- **Nonuniform** density
  - devices are located at random in the annulus at l_j according to a Poisson Point Process (PPP) with intensity ρ_j
  - **inverse-square law** for node density:

$$\frac{\rho_j}{\rho_{j-1}} = \frac{l_{j-1}^2}{l_j^{\,2}} \tag{4}$$

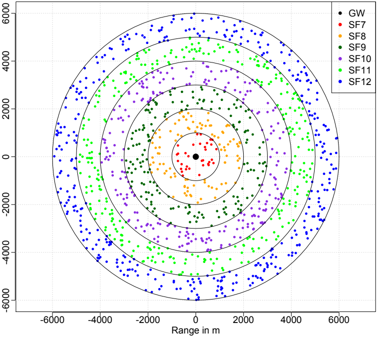
Nonuniform density

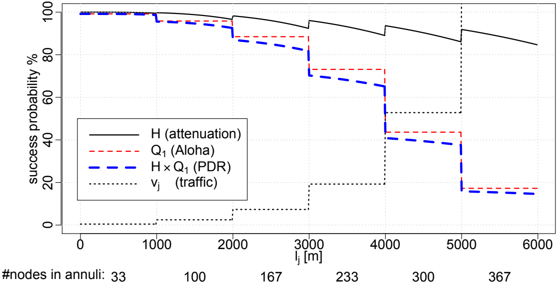




Inverse density and **equidistant SF annuli**, $n = 1200$.
809 nodes with PDR $> 80\%$.

Uniform density





Uniform density and **equidistant SF annuli**, $n = 1200$.
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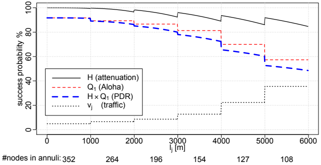
Part 3: LoRaWAN capacity for nonuniform node density



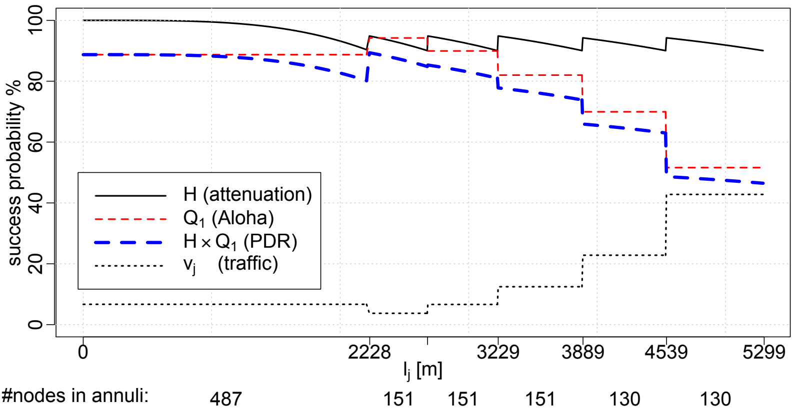
SF Allocation Strategies

1. **Equidistant** SF allocation with $l_{j+1} - l_j = l_0/6$
2. **SNR-based** SF allocation with $l_j = \{d : H(d) \geq \theta\}$
3. **PDR-based** SF allocation with $l_j = \{d : H(d) \times Q_1(d) \geq \theta\}$.

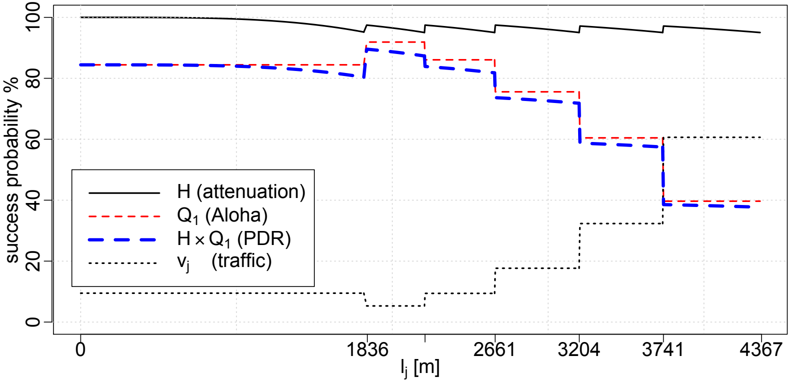
Nonuniform density

	SF7	SF8	SF9	SF10	SF11	SF12
	l_5	l_4	l_3	l_2	l_1	l_0
$H(l_j) = 90\%$	2.23	2.68	3.23	3.89	4.54	5.30
S_j/π [km^2]	4.96	2.23	3.24	4.69	5.49	7.47
ρ_j	1	0.69	0.48	0.33	0.24	0.18
$S_j/\pi \times \rho(l_j)$	4.96	1.54	1.54	1.54	1.32	1.32
$H(l_j) = 95\%$	1.84	2.21	2.66	3.20	3.74	4.37
S_j/π [km^2]	3.38	1.50	2.19	3.17	3.74	5.1
ρ_j	1	0.69	0.48	0.33	0.24	0.18
$S_j/\pi \times \rho(l_j)$	3.38	1.03	1.05	1.05	0.9	0.9
$H(l_j) = 99\%$	1.18	1.43	1.72	2.07	2.41	2.82
S_j/π [km^2]	1.40	0.63	0.91	1.33	1.55	2.11
ρ_j	1	0.69	0.48	0.33	0.24	0.18
$S_j/\pi \times \rho(l_j)$	1.40	0.44	0.44	0.44	0.37	0.37

SNR SF annuli $H(l_j) \geq 90\%$, N=1200, 787 nodes PDR $> 80\%$

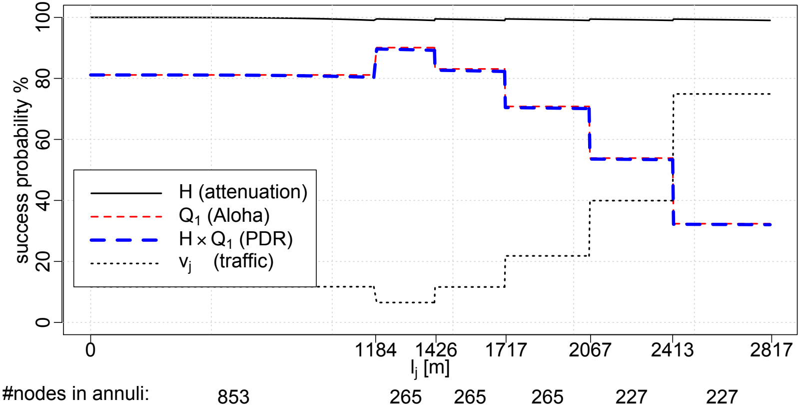


SNR SF annuli $H(l_j) \geq 95\%$, N=1700, 1115 nodes PDR $> 80\%$

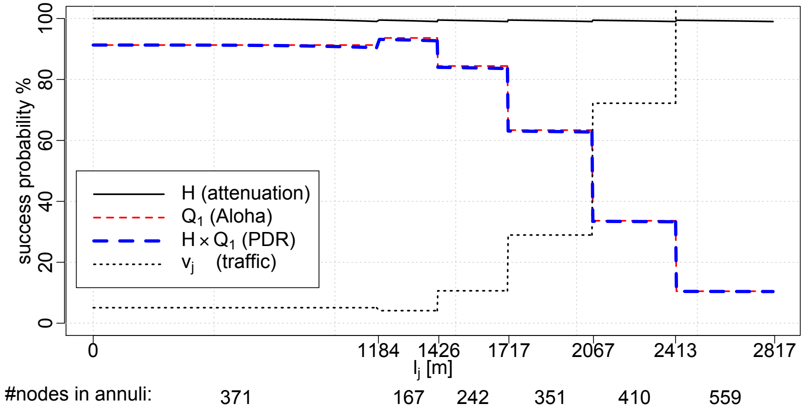


#nodes in annuli: 690 214 214 214 184 184

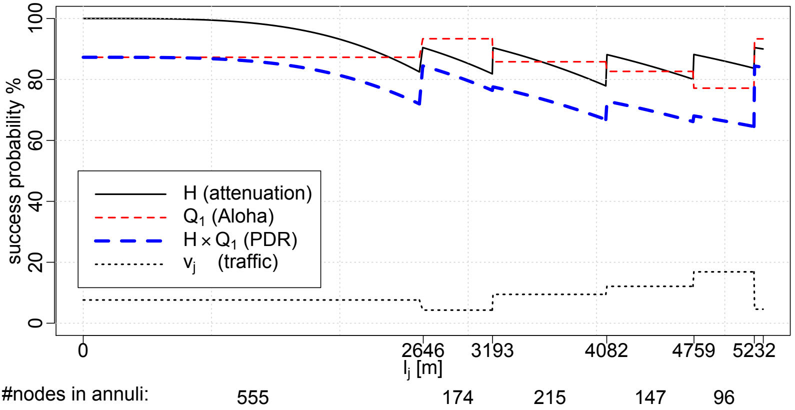
SNR SF annuli $H(l_j) \geq 99\%$, N=2100, 1377 nodes PDR $> 80\%$



Uniform density, SNR SF annuli $H(l_j) \geq 99\%$, N=2100, 776 nodes PDR $> 80\%$

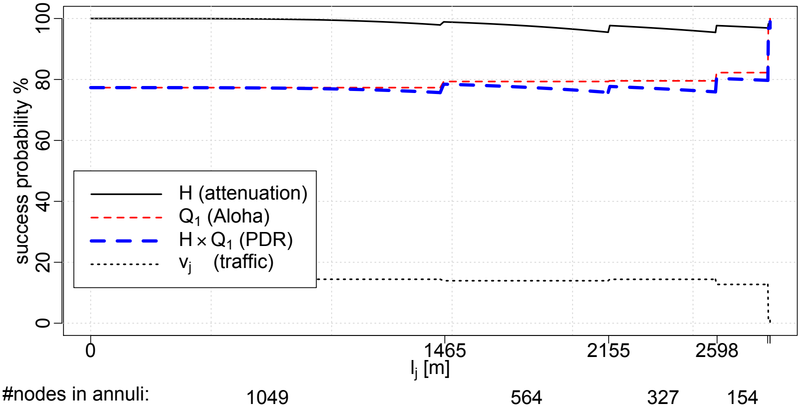


Inverse density, PDR SF annuli $H(l_j) \geq 90\%$, N=1200, 460 nodes PDR $> 80\%$

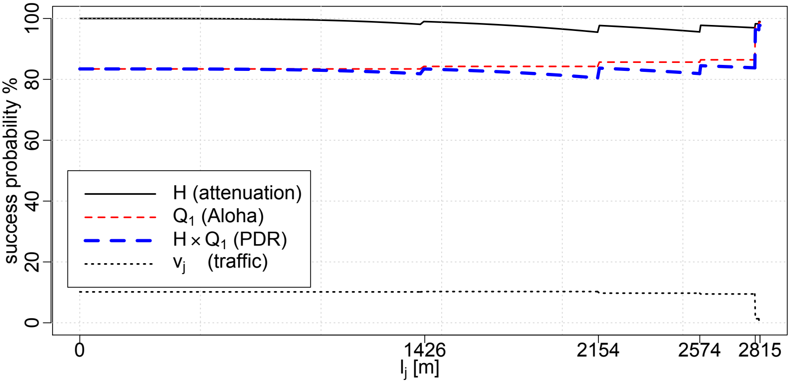


- adjust l_j with Nelder Mead simplex: $max(min(PDR(l_j, v_j)))$

Inverse density, PDR SF annuli $H(l_j) \geq 99\%$, N=2100, no nodes PDR $> 80\%$

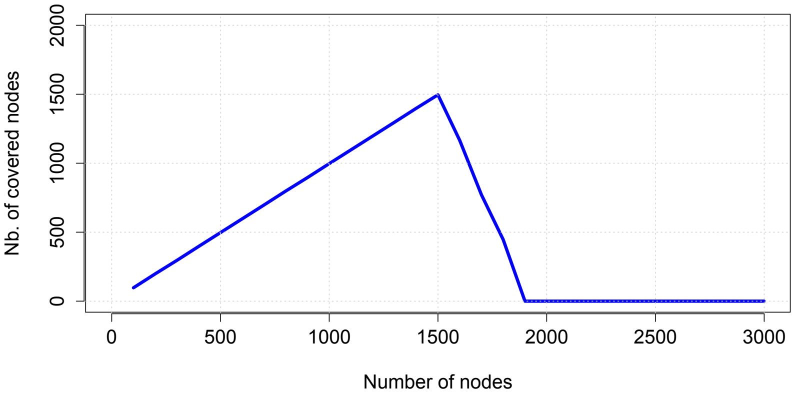


Inverse density, PDR SF annuli $H(l_j) \geq 99\%$, N=1500, 1500 PDR $> 80\%$

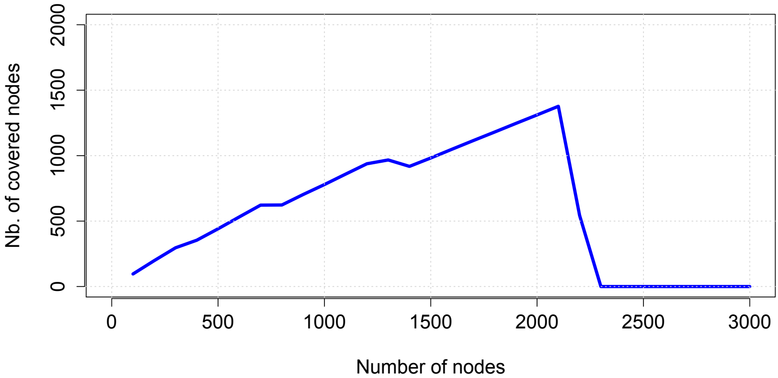


#nodes in annuli: 739 415 221 115

Number of nodes with PDR $> 80\%$, optimal allocation of l_j



Number of nodes with PDR $> 80\%$, SNR allocation of l_j



Conclusion

- The **smaller the radius**, the **more nodes** can be handled.
- For required PDR level and a target communication range $\Rightarrow$ we can find annuli I_j giving the maximal number of nodes that benefit from the PDR level.
- Natural trend towards configurations composed of **smaller cells** that concentrate nodes close to the gateway - nodes benefit from **low SF**, which also means **lower energy consumption**.
- To provide the required PDR level to more nodes, we need to consider **multiple gateways** that will increase the overall capacity while keeping low energy consumption.